
Fractal Dimension of the El Salvador Earthquake(2001) time Series

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1 Introduction

Earthquakes occur on the earth's surface as a result of rearrangement of terrestrial cortex or higher part of the mantle. The energy released in this process propagates over long distances in the form of elastic seismic waves [1]. In order to predict earthquakes many models have been proposed [2, 3]. Dynamics of an earthquake is so complicated that it is quite difficult to predict using available models. Seismicity is a classic example of a complex phenomenon that can be quantified using fractal concepts [4].

In this paper we have estimated the fractal dimension, maximum, as well as minimum of the singularities, and the half-width of the multifractal spectrum of the El Salvador Earthquake signal at different stations. The data has been taken from the California catalogue (http://nsmg.wr.usgs.gov/nsmn_eqdata.html). The paper has been arranged as follows: In section 2 the basic theory of multifractality has been discussed, and the results have been presented in section 3 .

2 Multifractal Analysis

The Hölder exponent of a time series $f(t)$ at the point t_0 is given by the largest exponent such that there exists a polynomial $P_n(t - t_0)$ of the order of n satisfying [5, 6, 7]

$$|f(t) - p_n(t - t_0)| \leq C|t - t_0|^{\alpha(t_0)} \quad (1)$$

The polynomial $P_n(t - t_0)$ corresponds to the Taylor series of $f(t)$ around $t = t_0$, up to n . The exponent α measures the irregularities of the function f . Higher positive value of $\alpha(t_0)$ indicates regularity in the function f . Negative α indicates spike in the signal. If $n < \alpha < n + 1$ it can be proved that the function f is n times differentiable but not $n + 1$ times at the point t_0 [8].

All the Hölder exponents present in the time series are given by the singularity spectrum $D(\alpha)$. This can be determined from the Wavelet Transform Modulus Maxima(WTMM). Before proceeding to find out the exponents α using wavelet analysis, we discuss about the wavelet transform.

2.1 Wavelet Analysis

In order to understand wavelet analysis, we have to first understand ‘*What is a wavelet?*’. A wavelet is a waveform of effectively limited duration that has an average value of zero, shown in the figure 1 (bottom). The difference of wavelets to sine waves, which are the basis of Fourier analysis, is that sinusoids do not have limited duration, but they extend from minus to plus

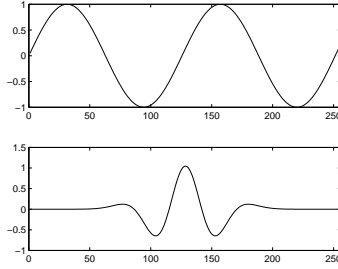


Fig. 1. A sinusoidal signal(up), and the gaussian Wavelet having four vanishing moments(bottom). Sinusoid has no limitation of duration where as wavelet has a limited duration.

infinity. And where sinusoids are smooth and predictable, wavelets tend to be irregular and asymmetric. For example, ‘gaus4’ wavelet (Fig. 1(bottom)) is defined as $\psi(t) = \frac{d^N}{dt^N} e^{-t^2/2}$, where $N=4$. Fourier analysis breaks up a signal into sine waves of various frequencies. Similarly, wavelet analysis breaks up a signal into shifted and scaled versions of the original (or mother) wavelet. It can be intuitively understood that signals with sharp changes might be better analyzed with an irregular wavelet than with a smooth sinusoid. Local features can be described better with wavelets that have local extent.

Wavelet transform can be defined as

$$Wf(s, b) = \frac{1}{s} \int_{-\infty}^{+\infty} f(t) \psi\left(\frac{t-b}{s}\right) dt \quad (2)$$

where s , and b are the scale and time respectively. In order to detect singularities we will further require ψ to be orthogonal to some low-order polynomials [8]:

$$\int_{-\infty}^{+\infty} t^m \psi(t) dt = 0, \quad \forall m, \quad 0 \leq m < N \quad (3)$$

for example, the wavelet in Figure 1 has four vanishing moments, i.e. $N=4$.

2.2 Singularity Detection

Since the wavelet has N vanishing moments, so $\int_{-\infty}^{+\infty} P_n(t - t_0)\psi(t) = 0$, (if $n < N$), and therefore, the wavelet coefficient only detects the singular part of the signal.

$$Wf(s, b) \sim s^{\alpha(t_0)}, \quad a \rightarrow 0^+, \quad (4)$$

So, as long as, $N > \alpha(t_0)$ the Hölder exponents can be extracted from log-log plot of the Equation 4 .

2.3 Wavelet Transform Modulus Maxima

Let $[u_p(s)]_{p \in \mathbb{Z}}$ be the position of all maxima of $|Wf(b, s)|$ at a fixed scale s . Then the partition function Z is defined as [11]

$$Z(q, s) = \sum_p |Wf(b, s)|^q \quad (5)$$

Z will be calculated from the WTMM. Drawing an analogy from thermodynamics, one can define the exponent $\tau(q)$ from the power law behavior of the partition function [9, 10, 11] as

$$Z(q, s) \sim a^{\tau(q)}, \quad a \rightarrow 0^+, \quad (6)$$

The log-log plot of Eqn 6 will give the $\tau(q)$ of the signal.

Now the multifractal spectrum $D(\alpha(q))$ vs $\alpha(q)$ can be computed from the Legendre transform

$$D(\alpha) = \min_q (q\alpha - \tau(q)) \quad (7)$$

where, the Hölder exponent $\alpha = \frac{\partial \tau(q)}{\partial q}$.

3 Results and Discussion

In the present paper we have analyzed the El Salvador earthquake data recorded at different stations as shown in the Table 1. In this table we have arranged the stations according to their distances from the epicenter.

Wavelet analysis of the data recorded at different stations shows that the major events of the earthquake have taken place at short time scales. For eg. Fig 2(top) shows a burst of activity in a short duration and the corresponding Continuous Wavelet Transform (CWT) in Fig 2(bottom) for the time series recorded at Santa Tacia station. In this figure(Fig 2[bottom]) the maximum correlation is shown by white color (which indicates maximum correlation), which occurs between 15 to 25 seconds approximately shown in fig 2. CWT of

Table 1. In this table the fractal dimension (3-rd column), minimum and maximum (4-th and 5-th column) values of the singularities have been shown for different stations according to their distance from the epicentral distance of the earthquake.

<i>Earthquake recording Station</i>	<i>Epicentral Distance(Km)</i>	Fractal Dim	Singularity(α)	
			α_{max}	α_{min}
Santiago de Maria	52.50648	0.81	2.23	1.46
Presa 15 De Septiembre Dam	63.85000	0.83	2.85	1.14
San Miguel	69.95400	0.88	2.85	1.68
Sensuntepeque	90.50100	0.84	2.53	1.40
Observatorio	91.02397	0.82	2.76	1.52
Cutuco	96.63410	0.84	2.60	1.48
Santa Tacia	97.99589	0.91	2.58	1.69
Acajutia Cepa	139.41800	0.89	3.05	1.83
Santa Ana	142.01300	0.86	3.32	1.49
Ahuachapan	157.35800	0.75	2.48	1.68
Cessa Metapan	165.78460	0.93	3.44	1.58

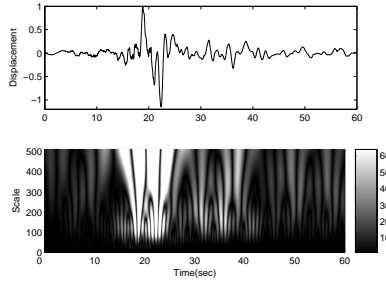


Fig. 2. Typical raw data recorded at station Santa Tacia (Top) and its Continuous wavelet transform(cwt)(bottom). From cwt it is clear that the major earthquake events occurs within few seconds (in between 15-20 sec).

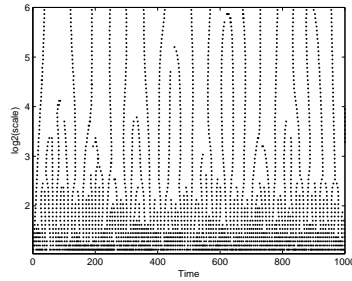


Fig. 3. WTMM skeleton of the data taken at Santa Tacia station(Raw data Fig 2).

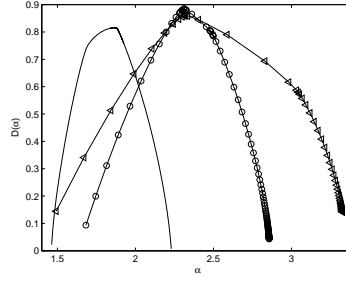


Fig. 4. Multifractal analysis analysis of El Salvador earthquake. In the above figure $-$, $-0-$, and $-◁-$, are the singularity spectrums of the data recorded at the stations Santiago de Maria, Santa Tacia, and Santa Ana respectively.

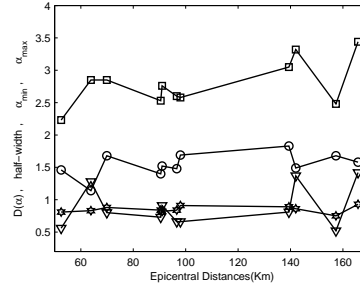


Fig. 5. From this figure it is clear that the fractal dimension of singularity support is around 1 ($- * -$), lower bound and upper bound of singularity increases with the epicentral distances ($-0-$, $-□-$ and respectively), and half-width has not such incremental behavior ($-▽-$).

the recorded data also shows that pseudo frequencies of the major events are less than 2 Hz. For Santa Tacia data it is few hundred mHz to 2 Hz. From the same figure (Fig 2[bottom]) it is also clear that the high frequencies i.e. 1-2 Hz come in very short range (1-4 seconds), and mHz frequencies comes with relatively longer durations (about 10 seconds). Multifractal analysis of the earthquake data recorded at different stations of increasing distances from the El Salvador earthquake epicenter of 2001 has been carried out. In the table 1 the first column represents the station according to their distance from the earthquake epicenter (distances shown in the second column is in km). In order to get the multifractal spectrum we first calculated the WTMM tree shown in the figure 3 as described in subsection 2.3. Using Legendre transform method we have obtained the multifractal spectrum shown in the figure 4. From multifractal analysis it is clear that the fractal dimension of the singularity support is around one. Lower bound and upper bound of the singularity increases with the distances of the station from the earthquake

epicenter shown in table 1 and in figure 4. It indicates the signal becomes smoother with distance, but the half width of the singularity support has random variation with distances.

In conclusion, the data shows a multifractal behavior, and the major event takes place in a short duration.

Acknowledgment

Some of the MATLAB function of Wavelab has been used in this analysis(address: <http://www-stat.stanford.edu/~wavelab>).

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